Analysis of Transient Electromagnetic Dipole

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Abstract: This paper presents the solution of a transient electromagnetic problem using COMSOL Multiphysics. The paper also presents a closed-form solution of a transient electromagnetic dipole. The computational solution compares well with a closed-form solution for this problem.

Keywords: Transient, Electromagnetics

1. Introduction

Propagation characteristics of an electromagnetic pulse generated by an electric dipole is used for a variety of applications such as geological mapping of mineral deposits in the ground or on the ocean floor, cellular or mobile communications and detection of unexploded ordnance. In all cases current is switched off and eddy currents are induced. The nature of the induced currents is a function of the transient waveform used to control switching of the current. However, the characteristics of the transient pulse when travelling in a conductive, dissipating medium are complex since the wave number is not linear and the dipole source creates a field that incorporates the near, intermediate and far fields. Consequently, the propagating pulse shifts from that of the excitation pulse and near field response to its spatial and time derivatives. This work focuses on implementing and demonstrating computational analysis of a transient electric dipole.

2. Analytical approach

This work implements Maxwell's equations in the RF module and optimizes solver parameter settings to resolve the transient nature of the pulse. The results of the analysis are compared with available analytical solutions for transient pulses.

The $x-$directed electric dipole moment $\mathbf{p}$ placed at the origin of coordinate system is defined by:

$$\mathbf{p} = \mathbf{e}_x I(t) ds \delta(\mathbf{r}) \quad (1)$$

where:

$\mathbf{e}_x$: unit vector of dipole axis

$I(t)$: transient current pulse

ds : dipole length

$\delta(\mathbf{r})$: Dirac delta function

A rectangular current pulse with a nonzero rise and decay time can be described by:

$$I(t) = \frac{1}{2t_1}\left\{ \begin{array}{l} \left(1 - e^{-\omega_p t}\right)H(t) - \\ \left[1 - e^{-\omega_p (t - 2t_1)}\right]H(t - 2t_1) \end{array} \right\} \quad (2)$$

where:

$H(t)$: Heaviside step function,

$2t_1$: width of the original rectangle pulse,

$\tau_p = \dfrac{1}{\omega_p}$: rise time which is taken to be equal to the decay time.

The electric field at plane $z = 0$ perpendicular to dipole axis is given in ref. [1] as:

$$E_x(\rho, t) = \frac{\mu_0 a I(t) ds}{16\pi\, t_1} \begin{cases} 0, \quad t = 0 \\ \mathsf{E}(\rho, t), \quad 0 < t < 2t_1 \\ \mathsf{E}(\rho, t) - \mathsf{E}(\rho, t - 2t_1), \quad t > 2t_1 \end{cases} \quad (3)$$

where:

$$\mathsf{E}(\rho, t) = -\frac{e^{-R^2}}{t\sqrt{2t}} \left\{ \begin{array}{l} \dfrac{1}{2R^3}\left[F(Z) - F(R)\right] + \\ \dfrac{\Omega}{R^2} G(Z) - \dfrac{2\Omega^2}{R} F(Z) \end{array} \right\} \quad (4)$$

$$F(Z) = \mathrm{Re}\left[e^{Z^2} erfc(Z)\right],$$
$$F(R) = \mathrm{Re}\left[e^{R^2} erfc(R)\right],$$
$$G(Z) = \mathrm{Im}\left[e^{Z^2} erfc(Z)\right],$$

$erfc(Z) = \dfrac{2}{\sqrt{\pi}} \displaystyle\int_Z^{\infty} e^{\lambda^2} d\lambda$ is complementary error function

$$Z = R + i\Omega, \qquad R = \frac{a\rho}{\sqrt{2t}}, \qquad \Omega = \sqrt{\omega_p t},$$
$$a = \sqrt{\frac{\mu_0 \sigma}{2}}, \ \rho = \sqrt{x^2 + y^2 + z^2} \qquad (6)$$

This solution is valid for the condition $\sigma >> \omega_p \varepsilon$

Implementation of the solution into COMSOL Multiphysics used the RF module to solve Maxwell's wave equations in terms of magnetic vector potential $\mathbf{A}$:

$$\mu\varepsilon \frac{\partial^2 \mathbf{A}}{\partial t^2} + \mu\sigma \frac{\partial \mathbf{A}}{\partial t} + \nabla \times (\nabla \times \mathbf{A}) = 0 \qquad (7)$$

Three symmetry planes are used to reduce the size of computational domain. Scattering boundary condition is imposed at the external boundary. Solver settings were optimized to resolve the transient nature of the pulse.

3. Results

The electric field resulting from application of a short, transient current pulse (Figure 1) to an electric dipole is given in Figure 2 for both the analytical and computational approaches. All results are obtained for media with dissipative media with electric conductivity $\sigma = 4 \ S/m$.

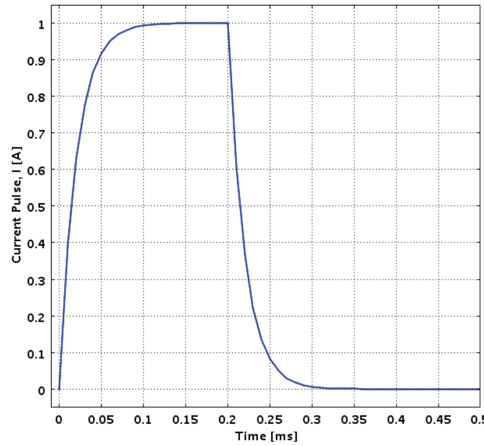

Figure 1. Transient current pulse applied to electric dipole. Pulse parameters: τ_1=0.1ms, ω_p=50kHz.

Excellent agreement is observed between the closed form analytical expression and the finite element results form COMSOL Multiphysics for propagation in a dissipative medium. The results show that although the applied current pulse extends over a period of 0.25ms, the electric field that is developed propagates for a period of 2.5ms. A visual representation of development of the field is given in Figures 3 and 4 for the short, transient pulse.

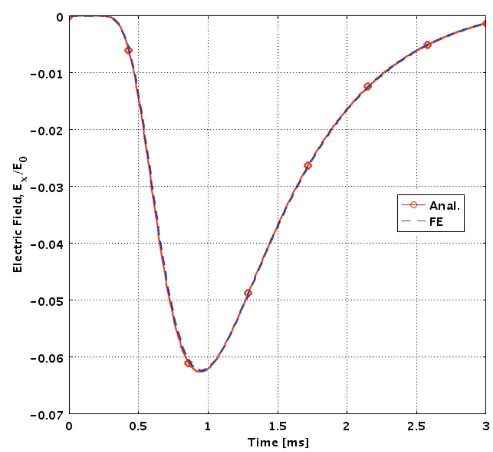

Figure 2: Electric field developed by transient current pulse with non-zero rise and decay time applied to an electric dipole. Normalization field is $E_0 = p / \left(\pi\sigma\rho^{3/2}\right)$. Observation point: x=0, y=50m, z=0.

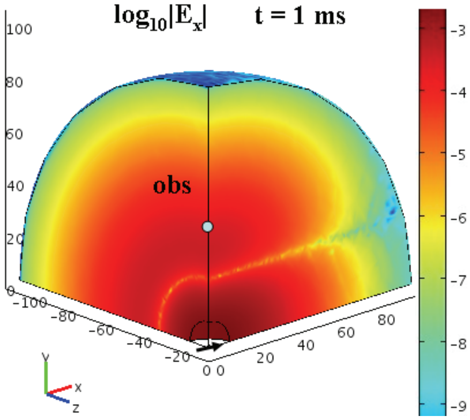

Figure 3: Electric field at t=1ms.

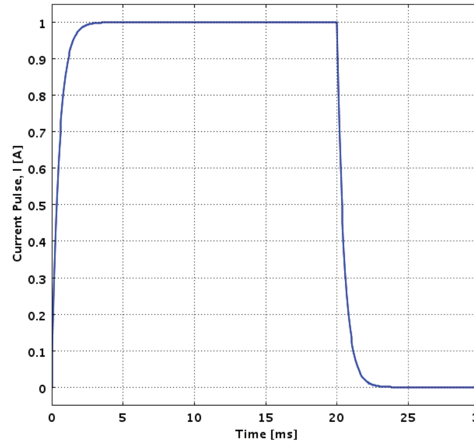

Figure 5: Applied current pulse with long duration. Pulse parameters: τ_1=10ms, ω_p=2kHz.

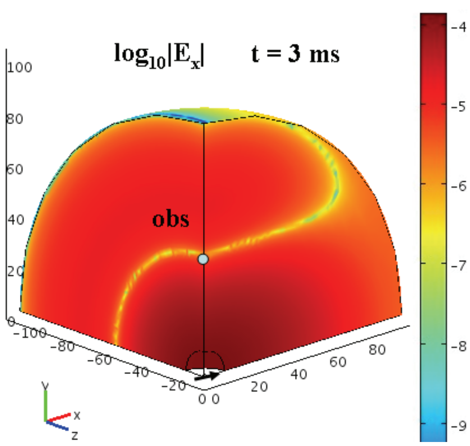

Figure 4: Electric field at t=3ms

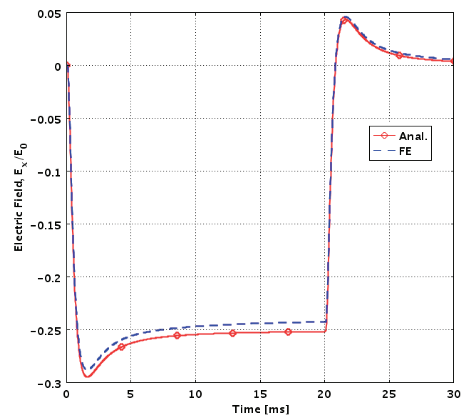

Figure 6: Electric field resulting from current pulse with long duration. Observation point: x=0, y=20m, z=0.

When a current pulse with longer duration, 25ms, is applied to the dipole (see Figure 5), the induced electric field shows different characteristics (see Figure 6).

Good agreement is once again observed between the analytical and computational results. The induced electric field is now dominated by the steady state nature of the applied current so that the electric field extends for a period of ~25ms. A visual representation of development of the field is given in Figures 7 and 8 for the short, transient pulse.

Application of a transient current pulse to an electric dipole changes the shape, amplitude, duration, and rise and decay times of a propagating electric wave in a dissipative medium. This arises because the wave number is not linear in the rise/decay frequency and the dipole has a response that is composed of the near, intermediate and far fields. Consequently, the form of the propagating pulse shifts from that of the excitation current and near field, to the spatial and time derivatives.

The electric field generated by a dipole excited by a pulse with finite, non-zero rise and decay time has a complicated behavior. The field consists of two transients each of which is the sum of two terms: a term that represents the response to the rectangular pulse with non-zero rise and decay time and a second term that accounts for the step discontinuity. Each transient contains the complete field and shifts from the form of the original pulse to the spatial and time derivatives.

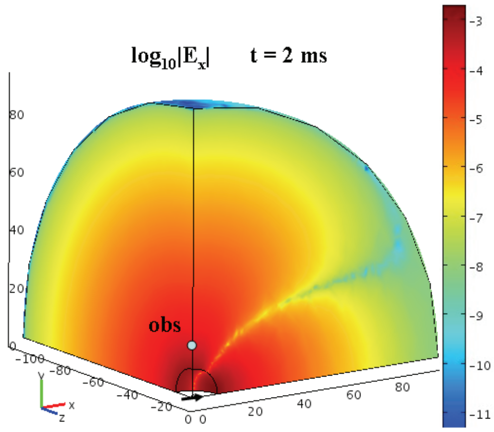

Figure 7: Electric filed due to long current pulse at t=2ms.

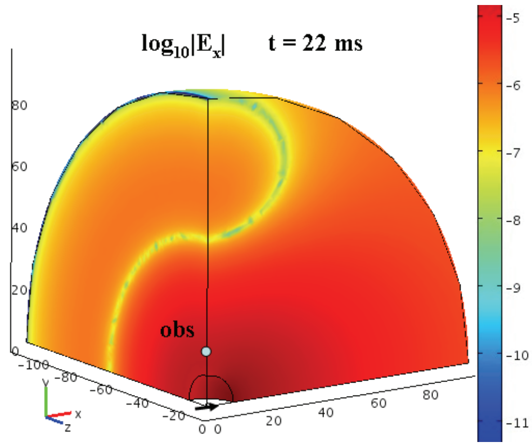

Figure 8: Electric filed due to long current pulse at t=22ms.

The results obtained here can be applied to many situations of electric field propagation in a highly dissipative medium.

4. Conclusions

These results clearly show the successful development of computational models to predict the electromagnetic field developed due to a transient pulse through an electric dipole. The analyses can be used to study the effects of properties of the dissipating medium and pulse characteristics on the electric fields that are generated.

8. References

1. D. Margetis, "Pulse Propagation in Sea Water," *J. Appl. Physics*, 1995, Vol. 77 (7), No. 1, pp. 2884-2888.